

JAMES STEWART | LOTHAR REDLIN | SALEEM WATSON



PRECALCULUS|6_e

MATHEMATICS FOR CALCULUS

EXPONENTS AND RADICALS

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$$

SPECIAL PRODUCTS

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

FACTORING FORMULAS

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$$|x| = a \text{ means } x = a \text{ or } x = -a.$$

$$|x| < a \text{ means } -a < x < a.$$

$$|x| > a \text{ means } x > a \text{ or } x < -a.$$

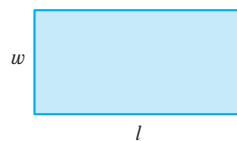
GEOMETRIC FORMULAS

Formulas for area A , perimeter P , circumference C , volume V :

Rectangle

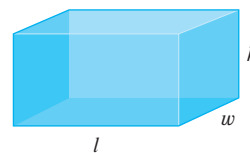
$$A = lw$$

$$P = 2l + 2w$$



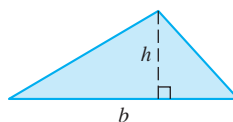
Box

$$V = lwh$$



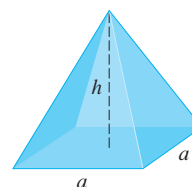
Triangle

$$A = \frac{1}{2}bh$$



Pyramid

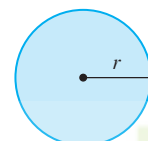
$$V = \frac{1}{3}ha^2$$



Circle

$$A = \pi r^2$$

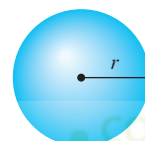
$$C = 2\pi r$$



Sphere

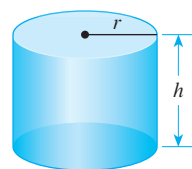
$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$



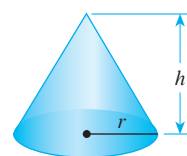
Cylinder

$$V = \pi r^2 h$$



Cone

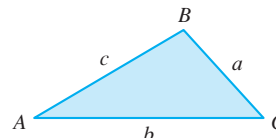
$$V = \frac{1}{3}\pi r^2 h$$



HERON'S FORMULA

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$



DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2 : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

LINES

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y -intercept b

$$y = mx + b$$

Two-intercept equation of line with x -intercept a and y -intercept b

$$\frac{x}{a} + \frac{y}{b} = 1$$

LOGARITHMS

$y = \log_a x$ means $a^y = x$

$$\log_a a^x = x$$

$$a^{\log_a x} = x$$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log x = \log_{10} x$$

$$\ln x = \log_e x$$

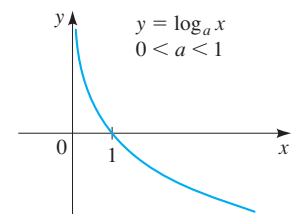
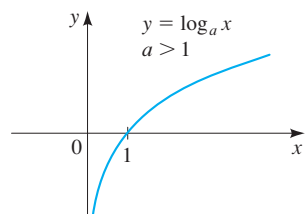
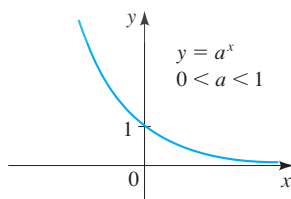
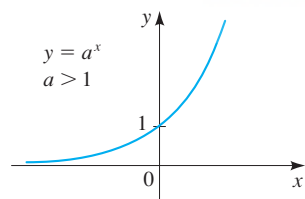
$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a x^b = b \log_a x$$

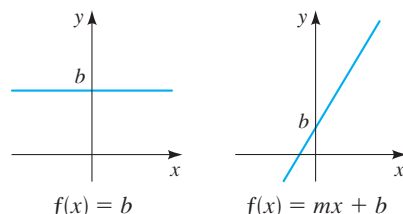
$$\log_b x = \frac{\log_a x}{\log_a b}$$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

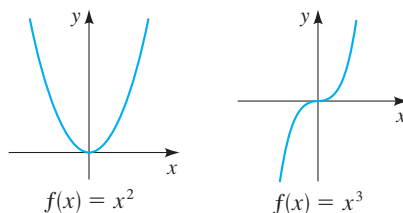


GRAPHS OF FUNCTIONS

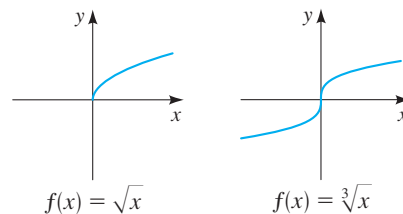
Linear functions: $f(x) = mx + b$



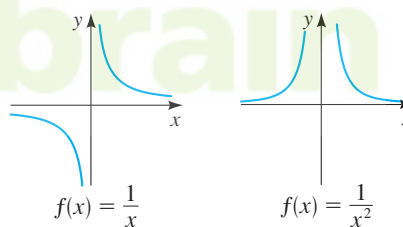
Power functions: $f(x) = x^n$



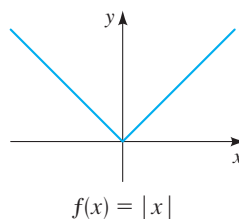
Root functions: $f(x) = \sqrt[n]{x}$



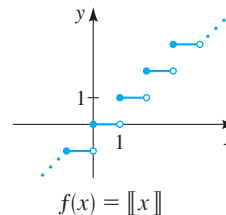
Reciprocal functions: $f(x) = 1/x^n$



Absolute value function



Greatest integer function



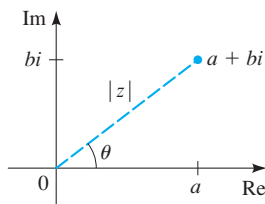
COMPLEX NUMBERS

For the complex number $z = a + bi$

the **conjugate** is $\bar{z} = a - bi$

the **modulus** is $|z| = \sqrt{a^2 + b^2}$

the **argument** is θ , where $\tan \theta = b/a$



Polar form of a complex number

For $z = a + bi$, the **polar form** is

$$z = r(\cos \theta + i \sin \theta)$$

where $r = |z|$ is the modulus of z and θ is the argument of z

De Moivre's Theorem

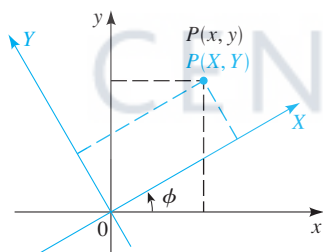
$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$$

$$\sqrt[n]{z} = [r(\cos \theta + i \sin \theta)]^{1/n}$$

$$= r^{1/n} \left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n} \right)$$

where $k = 0, 1, 2, \dots, n - 1$

ROTATION OF AXES



Rotation of axes formulas

$$x = X \cos \phi - Y \sin \phi$$

$$y = X \sin \phi + Y \cos \phi$$

Angle-of-rotation formula for conic sections

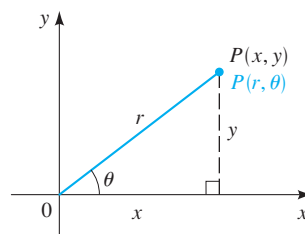
To eliminate the xy -term in the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

rotate the axis by the angle ϕ that satisfies

$$\cot 2\phi = \frac{A - C}{B}$$

POLAR COORDINATES



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

SUMS OF POWERS OF INTEGERS

$$\sum_{k=1}^n 1 = n$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

THE DERIVATIVE

The **average rate of change** of f between a and b is

$$\frac{f(b) - f(a)}{b - a}$$

The **derivative** of f at a is

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

AREA UNDER THE GRAPH OF f

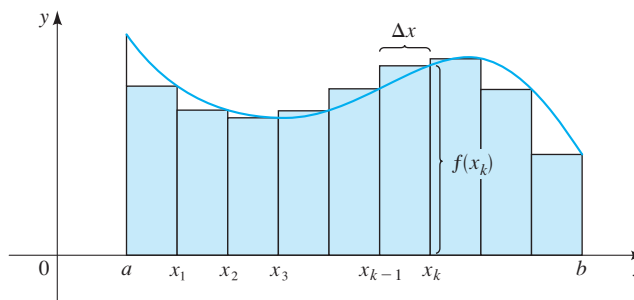
The **area under the graph of f** on the interval $[a, b]$ is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$$

where

$$\Delta x = \frac{b - a}{n}$$

$$x_k = a + k \Delta x$$



PRECALCULUS

MATHEMATICS FOR CALCULUS

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ABOUT THE AUTHORS

JAMES STEWART received his MS from Stanford University and his PhD from the University of Toronto. He did research at the University of London and was influenced by the famous mathematician George Polya at Stanford University. Stewart is Professor Emeritus at McMaster University and is currently Professor of Mathematics at the University of Toronto. His research field is harmonic analysis and the connections between mathematics and music. James Stewart is the author of a bestselling calculus textbook series published by Brooks/Cole, Cengage Learning, including *Calculus*, *Calculus: Early Transcendentals*, and *Calculus: Concepts and Contexts*; a series of pre-calculus texts; and a series of high-school mathematics textbooks.

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Stewart, Redlin, and Watson have also published *College Algebra*, *Trigonometry*, *Algebra and Trigonometry*, and (with Phyllis Panman) *College Algebra: Concepts and Contexts*.

ABOUT THE COVER

The cover photograph shows the Science Museum in the City of Arts and Sciences in Valencia, Spain, with a planetarium in the distance. Built from 1991 to 1996, it was designed by Santiago Calatrava, a Spanish architect. Calatrava has always been very interested in how mathematics can help him realize the buildings he imagines. As a young student, he taught himself descriptive geom-

etry from books in order to represent three-dimensional objects in two dimensions. Trained as both an engineer and an architect, he wrote a doctoral thesis in 1981 entitled "On the Foldability of Space Frames," which is filled with mathematics, especially geometric transformations. His strength as an engineer enables him to be daring in his architecture.

PRECALCULUS MATHEMATICS FOR CALCULUS

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PREFACE

What do students really need to know to be prepared for calculus? What tools do instructors really need to assist their students in preparing for calculus? These two questions have motivated the writing of this book.

To be prepared for calculus a student needs not only technical skill but also a clear understanding of concepts. Indeed, *conceptual understanding* and *technical skill* go hand in hand, each reinforcing the other. A student also needs to gain an appreciation for the power and utility of mathematics in *modeling* the real world. Every feature of this textbook is devoted to fostering these goals.

In writing this Sixth Edition our purpose is to further enhance the utility of the book as an instructional tool for teachers and as a learning tool for students. There are several major changes in this edition including a restructuring of each exercise set to better align the exercises with the examples of each section. In this edition each exercise set begins with *Concepts Exercises*, which encourage students to work with basic concepts and to use mathematical vocabulary appropriately. Several chapters have been reorganized and rewritten (as described below) to further focus the exposition on the main concepts; we have added a new chapter on vectors in two and three dimensions. In all these changes and numerous others (small and large) we have retained the main features that have contributed to the success of this book.

New to the Sixth Edition

- **Exercises** More than 20% of the exercises are new. This includes new *Concept Exercises* and new *Cumulative Review* exercises. Key exercises are now linked to examples in the text.
- **Book Companion Website** A new website www.stewartmath.com contains *Discovery Projects* for each chapter and *Focus on Problem Solving* sections that highlight different problem-solving principles outlined in the Prologue.
- **CHAPTER 2 Functions** This chapter has been completely rewritten to focus more sharply on the fundamental and crucial concept of *function*. The material on quadratic functions, formerly in this chapter, is now part of the chapter on polynomial functions.
- **CHAPTER 3 Polynomial and Rational Functions** This chapter now begins with a section on quadratic functions, leading to higher degree polynomial functions.
- **CHAPTER 4 Exponential and Logarithmic Functions** The material on the natural exponential function is now in a separate section.
- **CHAPTER 5 Trigonometric Functions: Unit Circle Approach** This chapter includes a new section on inverse trigonometric functions and their graphs. Introducing this topic here reinforces the function concept in the context of trigonometry.

- **CHAPTER 6 Trigonometric Functions: Right Triangle Approach** This chapter includes a new section on inverse trigonometric functions and right triangles (Section 6.4) which is needed in applying the Laws of Sines and Cosines in the following section, as well as for solving trigonometric equations in Chapter 7.
- **CHAPTER 7 Analytic Trigonometry** This chapter has been completely revised. There are two new sections on trigonometric equations (Sections 7.4 and 7.5). The material on this topic (formerly in Section 7.5) has been expanded and revised.
- **CHAPTER 8 Polar Coordinates and Parametric Equations** This chapter is now more sharply focused on the concept of a coordinate system. The section on parametric equations is new to this chapter. The material on vectors is now in its own chapter.
- **CHAPTER 9 Vectors in Two and Three Dimensions** This is a new chapter with a new *Focus on Modeling* section.
- **CHAPTER 10 Systems of Equations and Inequalities** The material on systems of nonlinear equations is now in a separate section.
- **CHAPTER 11 Conic Sections** This chapter is now more closely devoted to the topic of analytic geometry, especially the conic sections; the section on parametric equations has been moved to Chapter 8.

Teaching with the Help of This Book

We are keenly aware that good teaching comes in many forms, and that there are many different approaches to teaching the concepts and skills of precalculus. The organization of the topics in this book is designed to accommodate different teaching styles. For example, the trigonometry chapters have been organized so that either the unit circle approach or the right triangle approach can be taught first. Here are other special features that can be used to complement different teaching styles:

EXERCISE SETS The most important way to foster conceptual understanding and hone technical skill is through the problems that the instructor assigns. To that end we have provided a wide selection of exercises.

- **Concept Exercises** These exercises ask students to use mathematical language to state fundamental facts about the topics of each section.
- **Skills Exercises** Each exercise set is carefully graded, progressing from basic skill-development exercises to more challenging problems requiring synthesis of previously learned material with new concepts.
- **Applications Exercises** We have included substantial applied problems that we believe will capture the interest of students.
- **Discovery, Writing, and Group Learning** Each exercise set ends with a block of exercises labeled *Discovery* ■ *Discussion* ■ *Writing*. These exercises are designed to encourage students to experiment, preferably in groups, with the concepts developed in the section, and then to write about what they have learned, rather than simply look for the answer.
- **Now Try Exercise ...** At the end of each example in the text the student is directed to a similar exercise in the section that helps reinforce the concepts and skills developed in that example (see, for instance, page 4).
- **Check Your Answer** Students are encouraged to check whether an answer they obtained is reasonable. This is emphasized throughout the text in numerous *Check Your Answer* sidebars that accompany the examples. (See, for instance, page 52).

A COMPLETE REVIEW CHAPTER We have included an extensive review chapter primarily as a handy reference for the basic concepts that are preliminary to this course.

- **Chapter 1** This is the review chapter; it contains the fundamental concepts from algebra and analytic geometry that a student needs in order to begin a precalculus

course. As much or as little of this chapter can be covered in class as needed, depending on the background of the students.

- **Chapter 1 Test** The test at the end of Chapter 1 is designed as a diagnostic test for determining what parts of this review chapter need to be taught. It also serves to help students gauge exactly what topics they need to review.

FLEXIBLE APPROACH TO TRIGONOMETRY The trigonometry chapters of this text have been written so that either the right triangle approach or the unit circle approach may be taught first. Putting these two approaches in different chapters, each with its relevant applications, helps to clarify the purpose of each approach. The chapters introducing trigonometry are as follows:

- **Chapter 5 Trigonometric Functions: Unit Circle Approach** This chapter introduces trigonometry through the unit circle approach. This approach emphasizes that the trigonometric functions are functions of real numbers, just like the polynomial and exponential functions with which students are already familiar.
- **Chapter 6 Trigonometric Functions: Right Triangle Approach** This chapter introduces trigonometry through the right triangle approach. This approach builds on the foundation of a conventional high-school course in trigonometry.

Another way to teach trigonometry is to intertwine the two approaches. Some instructors teach this material in the following order: Sections 5.1, 5.2, 6.1, 6.2, 6.3, 5.3, 5.4, 5.5, 5.6, 6.4, 6.5, and 6.6. Our organization makes it easy to do this without obscuring the fact that the two approaches involve distinct representations of the same functions.

GRAPHING CALCULATORS AND COMPUTERS We make use of graphing calculators and computers in examples and exercises throughout the book. Our calculator-oriented examples are always preceded by examples in which students must graph or calculate by hand, so that they can understand precisely what the calculator is doing when they later use it to simplify the routine, mechanical part of their work. The graphing calculator sections, subsections, examples, and exercises, all marked with the special symbol , are optional and may be omitted without loss of continuity. We use the following capabilities of the calculator.

- **Graphing, Regression, Matrix Algebra** The capabilities of the graphing calculator are used throughout the text to graph and analyze functions, families of functions, and sequences; to calculate and graph regression curves; to perform matrix algebra; to graph linear inequalities; and other powerful uses.
- **Simple Programs** We exploit the programming capabilities of a graphing calculator to simulate real-life situations, to sum series, or to compute the terms of a recursive sequence. (See, for instance, pages 787 and 791.)

FOCUS ON MODELING The “modeling” theme has been used throughout to unify and clarify the many applications of precalculus. We have made a special effort to clarify the essential process of translating problems from English into the language of mathematics (see pages 214 and 636).

- **Constructing Models** There are numerous applied problems throughout the book where students are given a model to analyze (see, for instance, page 228). But the material on modeling, in which students are required to *construct* mathematical models, has been organized into clearly defined sections and subsections (see for example, pages 213, 340, and 427).
- **Focus on Modeling** Each chapter concludes with a *Focus on Modeling* section. The first such section, after Chapter 1, introduces the basic idea of modeling a real-life situation by fitting lines to data (linear regression). Other sections present ways in which polynomial, exponential, logarithmic, and trigonometric functions, and systems of inequalities can all be used to model familiar phenomena from the sciences and from everyday life (see for example pages 296, 357, and 427).

BOOK COMPANION WEBSITE A website that accompanies this book is located at www.stewartmath.com. The site includes many useful resources for teaching precalculus, including the following:

- **Discovery Projects** *Discovery Projects* for each chapter are available on the website. Each project provides a challenging but accessible set of activities that enable students (perhaps working in groups) to explore in greater depth an interesting aspect of the topic they have just learned. (See for instance the *Discovery Projects Visualizing a Formula, Relations and Functions, Will the Species Survive?*, and *Computer Graphics I and II*.)
- **Focus on Problem Solving** Several *Focus on Problem Solving* sections are available on the website. Each such section highlights one of the problem-solving principles introduced in the Prologue and includes several challenging problems. (See for instance *Recognizing Patterns, Using Analogy, Introducing Something Extra, Taking Cases*, and *Working Backward*.)

MATHEMATICAL VIGNETTES Throughout the book we make use of the margins to provide historical notes, key insights, or applications of mathematics in the modern world. These serve to enliven the material and show that mathematics is an important, vital activity, and that even at this elementary level it is fundamental to everyday life.

- **Mathematical Vignettes** These vignettes include biographies of interesting mathematicians and often include a key insight that the mathematician discovered and which is relevant to precalculus. (See, for instance, the vignettes on Viète, page 49; Salt Lake City, page 84; and radiocarbon dating, page 333).
- **Mathematics in the Modern World** This is a series of vignettes that emphasizes the central role of mathematics in current advances in technology and the sciences (see pages 283, 700, and 759, for example).

REVIEW SECTIONS AND CHAPTER TESTS Each chapter ends with an extensive review section including the following.

- **Concept Check** The *Concept Check* at the end of each chapter is designed to get the students to think about and explain in their own words the ideas presented in the chapter. These can be used as writing exercises, in a classroom discussion setting, or for personal study.
- **Review Exercises** The *Review Exercises* at the end of each chapter recapitulate the basic concepts and skills of the chapter and include exercises that combine the different ideas learned in the chapter.
- **Chapter Test** The review sections conclude with a *Chapter Test* designed to help students gauge their progress.
- **Cumulative Review Tests** The *Cumulative Review Tests* following Chapters 4, 7, 9, 11, and 13 combine skills and concepts from the preceding chapters and are designed to highlight the connections between the topics in these related chapters.
- **Answers** Brief answers to odd-numbered exercises in each section (including the review exercises), and to all questions in the Concepts Exercises and Chapter Tests, are given in the back of the book.

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Book Companion Website

A new website **www.stewartmath.com** contains *Discovery Projects* for each chapter and *Focus on Problem Solving* sections that highlight different problem-solving principles outlined in the Prologue.

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Visit **www.cengagebrain.com** to access additional course materials and companion resources. At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where free companion resources can be found.

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The Text-Specific DVDs include new learning objective based lecture videos. These DVDs provide comprehensive coverage of the course—along with additional explanations of concepts, sample problems, and applications—to help students review essential topics.

This textbook was written for you to use as a guide to mastering precalculus mathematics. Here are some suggestions to help you get the most out of your course.

First of all, you should read the appropriate section of text *before* you attempt your homework problems. Reading a mathematics text is quite different from reading a novel, a newspaper, or even another textbook. You may find that you have to reread a passage several times before you understand it. Pay special attention to the examples, and work them out yourself with pencil and paper as you read. Then do the linked exercises referred to in “*Now Try Exercise . . .*” at the end of each example. With this kind of preparation you will be able to do your homework much more quickly and with more understanding.

Don’t make the mistake of trying to memorize every single rule or fact you may come across. Mathematics doesn’t consist simply of memorization. Mathematics is a *problem-solving art*, not just a collection of facts. To master the subject you must solve problems—lots of problems. Do as many of the exercises as you can. Be sure to write your solutions in a logical, step-by-step fashion. Don’t give up on a problem if you can’t solve it right away. Try to understand the problem more clearly—reread it thoughtfully and relate it to what you have learned from your teacher and from the examples in the text. Struggle with it until you solve it. Once you have done this a few times you will begin to understand what mathematics is really all about.

Answers to the odd-numbered exercises, as well as all the answers to each chapter test, appear at the back of the book. If your answer differs from the one given, don’t immediately assume that you are wrong. There may be a calculation that connects the two answers and makes both correct. For example, if you get $1/(\sqrt{2} - 1)$ but the answer given is $1 + \sqrt{2}$, your answer *is* correct, because you can multiply both numerator and denominator of your answer by $\sqrt{2} + 1$ to change it to the given answer. In rounding approximate answers, follow the guidelines in the Appendix: *Calculations and Significant Figures*.

The symbol  is used to warn against committing an error. We have placed this symbol in the margin to point out situations where we have found that many of our students make the same mistake.

ABBREVIATIONS

cm	centimeter	mg	milligram
dB	decibel	MHz	megahertz
F	farad	mi	mile
ft	foot	min	minute
g	gram	mL	milliliter
gal	gallon	mm	millimeter
h	hour	N	Newton
H	henry	qt	quart
Hz	Hertz	oz	ounce
in.	inch	s	second
J	Joule	Ω	ohm
kcal	kilocalorie	V	volt
kg	kilogram	W	watt
km	kilometer	yd	yard
kPa	kilopascal	yr	year
L	liter	°C	degree Celsius
lb	pound	°F	degree Fahrenheit
lm	lumen	K	Kelvin
M	mole of solute	⇒	implies
	per liter of solution	⇔	is equivalent to
m	meter		

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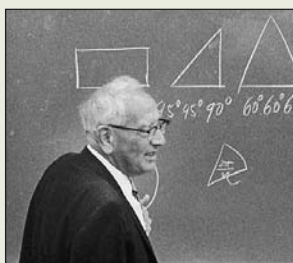
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PROLOGUE PRINCIPLES OF PROBLEM SOLVING

Chuck Painter/Stanford News Service



GEORGE POLYA (1887–1985) is famous among mathematicians for his ideas on problem solving. His lectures on problem solving at Stanford University attracted overflow crowds whom he held on the edges of their seats, leading them to discover solutions for themselves. He was able to do this because of his deep insight into the psychology of problem solving. His well-known book *How To Solve It* has been translated into 15 languages. He said that Euler (see page 266) was unique among great mathematicians because he explained how he found his results. Polya often said to his students and colleagues, “Yes, I see that your proof is correct, but how did you discover it?” In the preface to *How To Solve It*, Polya writes, “A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.”

The ability to solve problems is a highly prized skill in many aspects of our lives; it is certainly an important part of any mathematics course. There are no hard and fast rules that will ensure success in solving problems. However, in this Prologue we outline some general steps in the problem-solving process and we give principles that are useful in solving certain problems. These steps and principles are just common sense made explicit. They have been adapted from George Polya’s insightful book *How To Solve It*.

1. Understand the Problem

The first step is to read the problem and make sure that you understand it. Ask yourself the following questions:

What is the unknown?
What are the given quantities?
What are the given conditions?

For many problems it is useful to

draw a diagram

and identify the given and required quantities on the diagram. Usually, it is necessary to

introduce suitable notation

In choosing symbols for the unknown quantities, we often use letters such as a , b , c , m , n , x , and y , but in some cases it helps to use initials as suggestive symbols, for instance, V for volume or t for time.

2. Think of a Plan

Find a connection between the given information and the unknown that enables you to calculate the unknown. It often helps to ask yourself explicitly: “How can I relate the given to the unknown?” If you don’t see a connection immediately, the following ideas may be helpful in devising a plan.

► Try to Recognize Something Familiar

Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown.

► Try to Recognize Patterns

Certain problems are solved by recognizing that some kind of pattern is occurring. The pattern could be geometric, numerical, or algebraic. If you can see regularity or repetition in a problem, then you might be able to guess what the pattern is and then prove it.

► Use Analogy

Try to think of an analogous problem, that is, a similar or related problem but one that is easier than the original. If you can solve the similar, simpler problem, then it might give you the clues you need to solve the original, more difficult one. For instance, if a problem involves very large numbers, you could first try a similar problem with smaller numbers. Or if the problem is in three-dimensional geometry, you could look for something similar in two-dimensional geometry. Or if the problem you start with is a general one, you could first try a special case.

► Introduce Something Extra

You might sometimes need to introduce something new—an auxiliary aid—to make the connection between the given and the unknown. For instance, in a problem for which a diagram is useful, the auxiliary aid could be a new line drawn in the diagram. In a more algebraic problem the aid could be a new unknown that relates to the original unknown.

► Take Cases

You might sometimes have to split a problem into several cases and give a different argument for each case. For instance, we often have to use this strategy in dealing with absolute value.

► Work Backward

Sometimes it is useful to imagine that your problem is solved and work backward, step by step, until you arrive at the given data. Then you might be able to reverse your steps and thereby construct a solution to the original problem. This procedure is commonly used in solving equations. For instance, in solving the equation $3x - 5 = 7$, we suppose that x is a number that satisfies $3x - 5 = 7$ and work backward. We add 5 to each side of the equation and then divide each side by 3 to get $x = 4$. Since each of these steps can be reversed, we have solved the problem.

► Establish Subgoals

In a complex problem it is often useful to set subgoals (in which the desired situation is only partially fulfilled). If you can attain or accomplish these subgoals, then you might be able to build on them to reach your final goal.

► Indirect Reasoning

Sometimes it is appropriate to attack a problem indirectly. In using **proof by contradiction** to prove that P implies Q , we assume that P is true and Q is false and try to see why this cannot happen. Somehow we have to use this information and arrive at a contradiction to what we absolutely know is true.

► Mathematical Induction

In proving statements that involve a positive integer n , it is frequently helpful to use the Principle of Mathematical Induction, which is discussed in Section 12.5.

3. Carry Out the Plan

In Step 2, a plan was devised. In carrying out that plan, you must check each stage of the plan and write the details that prove that each stage is correct.

4. Look Back

Having completed your solution, it is wise to look back over it, partly to see whether any errors have been made and partly to see whether you can discover an easier way to solve the problem. Looking back also familiarizes you with the method of solution, which may be useful for solving a future problem. Descartes said, “Every problem that I solved became a rule which served afterwards to solve other problems.”

We illustrate some of these principles of problem solving with an example.

PROBLEM | Average Speed

A driver sets out on a journey. For the first half of the distance, she drives at the leisurely pace of 30 mi/h; during the second half she drives 60 mi/h. What is her average speed on this trip?

THINKING ABOUT THE PROBLEM

It is tempting to take the average of the speeds and say that the average speed for the entire trip is

$$\frac{30 + 60}{2} = 45 \text{ mi/h}$$

But is this simple-minded approach really correct?

Let's look at an easily calculated special case. Suppose that the total distance traveled is 120 mi. Since the first 60 mi is traveled at 30 mi/h, it takes 2 h. The second 60 mi is traveled at 60 mi/h, so it takes one hour. Thus, the total time is $2 + 1 = 3$ hours and the average speed is

$$\frac{120}{3} = 40 \text{ mi/h}$$

So our guess of 45 mi/h was wrong.

SOLUTION

Understand the problem ▶ We need to look more carefully at the meaning of average speed. It is defined as

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

Introduce notation ▶ Let d be the distance traveled on each half of the trip. Let t_1 and t_2 be the times taken for the first and second halves of the trip. Now we can write down the information we have been given. For the first half of the trip we have

$$30 = \frac{d}{t_1}$$

and for the second half we have

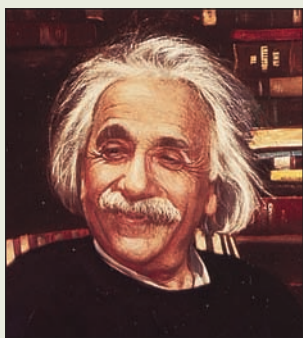
$$60 = \frac{d}{t_2}$$

Identify the unknown ▶ Now we identify the quantity that we are asked to find:

$$\text{average speed for entire trip} = \frac{\text{total distance}}{\text{total time}} = \frac{2d}{t_1 + t_2}$$

Connect the given with the unknown ▶ To calculate this quantity, we need to know t_1 and t_2 , so we solve the above equations for these times:

$$t_1 = \frac{d}{30} \quad t_2 = \frac{d}{60}$$



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Don't feel bad if you can't solve these problems right away. Problems 1 and 4 were sent to Albert Einstein by his friend Wertheimer. Einstein (and his friend Bucky) enjoyed the problems and wrote back to Wertheimer. Here is part of his reply:

Your letter gave us a lot of amusement. The first intelligence test fooled both of us (Bucky and me). Only on working it out did I notice that no time is available for the downhill run! Mr. Bucky was also taken in by the second example, but I was not. Such drolleries show us how stupid we are!

(See *Mathematical Intelligencer*, Spring 1990, page 41.)

Now we have the ingredients needed to calculate the desired quantity:

$$\begin{aligned}\text{average speed} &= \frac{2d}{t_1 + t_1} = \frac{2d}{\frac{d}{30} + \frac{d}{60}} \\ &= \frac{60(2d)}{60\left(\frac{d}{30} + \frac{d}{60}\right)} \quad \text{Multiply numerator and denominator by 60} \\ &= \frac{120d}{2d + d} = \frac{120d}{3d} = 40\end{aligned}$$

So the average speed for the entire trip is 40 mi/h.

PROBLEMS

- Distance, Time, and Speed** An old car has to travel a 2-mile route, uphill and down. Because it is so old, the car can climb the first mile—the ascent—no faster than an average speed of 15 mi/h. How fast does the car have to travel the second mile—on the descent it can go faster, of course—to achieve an average speed of 30 mi/h for the trip?
- Comparing Discounts** Which price is better for the buyer, a 40% discount or two successive discounts of 20%?
- Cutting up a Wire** A piece of wire is bent as shown in the figure. You can see that one cut through the wire produces four pieces and two parallel cuts produce seven pieces. How many pieces will be produced by 142 parallel cuts? Write a formula for the number of pieces produced by n parallel cuts.



- Amoeba Propagation** An amoeba propagates by simple division; each split takes 3 minutes to complete. When such an amoeba is put into a glass container with a nutrient fluid, the container is full of amoebas in one hour. How long would it take for the container to be filled if we start with not one amoeba, but two?
- Batting Averages** Player A has a higher batting average than player B for the first half of the baseball season. Player A also has a higher batting average than player B for the second half of the season. Is it necessarily true that player A has a higher batting average than player B for the entire season?
- Coffee and Cream** A spoonful of cream is taken from a pitcher of cream and put into a cup of coffee. The coffee is stirred. Then a spoonful of this mixture is put into the pitcher of cream. Is there now more cream in the coffee cup or more coffee in the pitcher of cream?
- Wrapping the World** A ribbon is tied tightly around the earth at the equator. How much more ribbon would you need if you raised the ribbon 1 ft above the equator everywhere? (You don't need to know the radius of the earth to solve this problem.)
- Ending Up Where You Started** A woman starts at a point P on the earth's surface and walks 1 mi south, then 1 mi east, then 1 mi north, and finds herself back at P , the starting point. Describe all points P for which this is possible. [Hint: There are infinitely many such points, all but one of which lie in Antarctica.]



Many more problems and examples that highlight different problem-solving principles are available at the book companion website: www.stewartmath.com. You can try them as you progress through the book.

APPENDIX Calculations and Significant Figures

Most of the applied examples and exercises in this book involve approximate values. For example, one exercise states that the moon has a radius of 1074 miles. This does not mean that the moon's radius is exactly 1074 miles but simply that this is the radius rounded to the nearest mile.

One simple method for specifying the accuracy of a number is to state how many **significant digits** it has. The significant digits in a number are the ones from the first nonzero digit to the last nonzero digit (reading from left to right). Thus, 1074 has four significant digits, 1070 has three, 1100 has two, and 1000 has one significant digit. This rule may sometimes lead to ambiguities. For example, if a distance is 200 km to the nearest kilometer, then the number 200 really has three significant digits, not just one. This ambiguity is avoided if we use scientific notation—that is, if we express the number as a multiple of a power of 10:

$$2.00 \times 10^2$$

When working with approximate values, students often make the mistake of giving a final answer with *more* significant digits than the original data. This is incorrect because you cannot “create” precision by using a calculator. The final result can be no more accurate than the measurements given in the problem. For example, suppose we are told that the two shorter sides of a right triangle are measured to be 1.25 and 2.33 inches long. By the Pythagorean Theorem, we find, using a calculator, that the hypotenuse has length

$$\sqrt{1.25^2 + 2.33^2} \approx 2.644125564 \text{ in.}$$

But since the given lengths were expressed to three significant digits, the answer cannot be any more accurate. We can therefore say only that the hypotenuse is 2.64 in. long, rounding to the nearest hundredth.

In general, the final answer should be expressed with the same accuracy as the *least*-accurate measurement given in the statement of the problem. The following rules make this principle more precise.

RULES FOR WORKING WITH APPROXIMATE DATA

1. When multiplying or dividing, round off the final result so that it has as many *significant digits* as the given value with the fewest number of significant digits.
2. When adding or subtracting, round off the final result so that it has its last significant digit in the *decimal place* in which the least-accurate given value has its last significant digit.
3. When taking powers or roots, round off the final result so that it has the same number of *significant digits* as the given value.

As an example, suppose that a rectangular table top is measured to be 122.64 in. by 37.3 in. We express its area and perimeter as follows:

$$\text{Area} = \text{length} \times \text{width} = 122.64 \times 37.3 \approx 4570 \text{ in}^2$$

Three significant digits

$$\text{Perimeter} = 2(\text{length} + \text{width}) = 2(122.64 + 37.3) \approx 319.9 \text{ in.}$$

Tenths digit



Note that in the formula for the perimeter, the value 2 is an exact value, not an approximate measurement. It therefore does not affect the accuracy of the final result. In general, if a problem involves only exact values, we may express the final answer with as many significant digits as we wish.

Note also that to make the final result as accurate as possible, *you should wait until the last step to round off your answer*. If necessary, use the memory feature of your calculator to retain the results of intermediate calculations.



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SEQUENCES AND SERIES

Arithmetic

$$a, a + d, a + 2d, a + 3d, a + 4d, \dots$$

$$a_n = a + (n - 1)d$$

$$S_n = \sum_{k=1}^n a_k = \frac{n}{2} [2a + (n - 1)d] = n \left(\frac{a + a_n}{2} \right)$$

Geometric

$$a, ar, ar^2, ar^3, ar^4, \dots$$

$$a_n = ar^{n-1}$$

$$S_n = \sum_{k=1}^n a_k = a \frac{1 - r^n}{1 - r}$$

If $|r| < 1$, then the sum of an infinite geometric series is

$$S = \frac{a}{1 - r}$$

THE BINOMIAL THEOREM

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

FINANCE

Compound interest

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

where A is the amount after t years, P is the principal, r is the interest rate, and the interest is compounded n times per year.

Amount of an annuity

$$A_f = R \frac{(1 + i)^n - 1}{i}$$

where A_f is the final amount, i is the interest rate per time period, and there are n payments of size R .

Present value of an annuity

$$A_p = R \frac{1 - (1 + i)^{-n}}{i}$$

where A_p is the present value, i is the interest rate per time period, and there are n payments of size R .

Installment buying

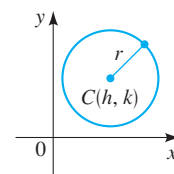
$$R = \frac{iA_p}{1 - (1 + i)^{-n}}$$

where R is the size of each payment, i is the interest rate per time period, A_p is the amount of the loan, and n is the number of payments.

CONIC SECTIONS

Circles

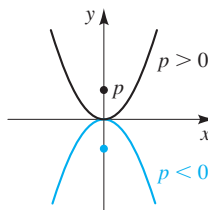
$$(x - h)^2 + (y - k)^2 = r^2$$



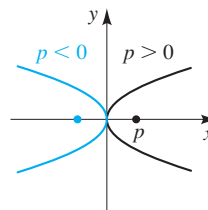
Parabolas

$$x^2 = 4py$$

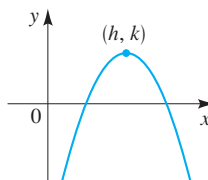
$$y^2 = 4px$$



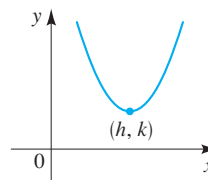
Focus $(0, p)$, directrix $y = -p$



Focus $(p, 0)$, directrix $x = -p$



$$y = a(x - h)^2 + k, \\ a < 0, \quad h > 0, \quad k > 0$$

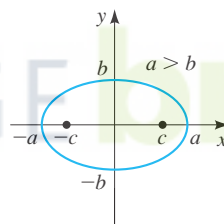


$$y = a(x - h)^2 + k, \\ a > 0, \quad h > 0, \quad k > 0$$

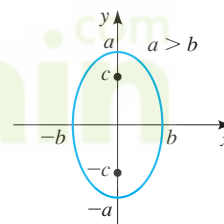
Ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

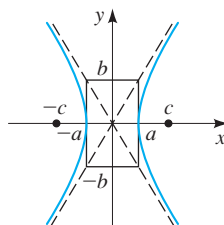


Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

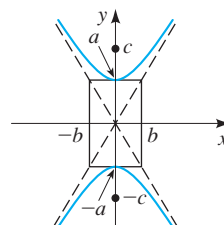
Hyperbolas

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$-\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

ANGLE MEASUREMENT

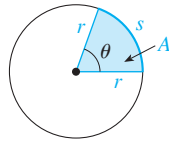
$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \quad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta \quad A = \frac{1}{2}r^2\theta \quad (\theta \text{ in radians})$$

To convert from degrees to radians, multiply by $\frac{\pi}{180}$.

To convert from radians to degrees, multiply by $\frac{180}{\pi}$.

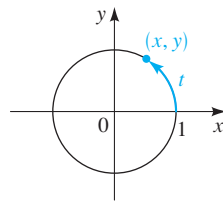


TRIGONOMETRIC FUNCTIONS OF REAL NUMBERS

$$\sin t = y \quad \csc t = \frac{1}{y}$$

$$\cos t = x \quad \sec t = \frac{1}{x}$$

$$\tan t = \frac{y}{x} \quad \cot t = \frac{x}{y}$$

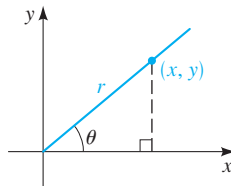


TRIGONOMETRIC FUNCTIONS OF ANGLES

$$\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

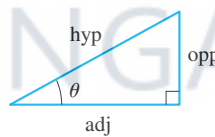


RIGHT ANGLE TRIGONOMETRY

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}}$$

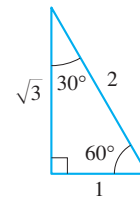
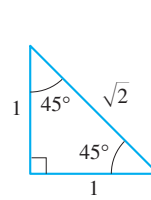
$$\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}$$



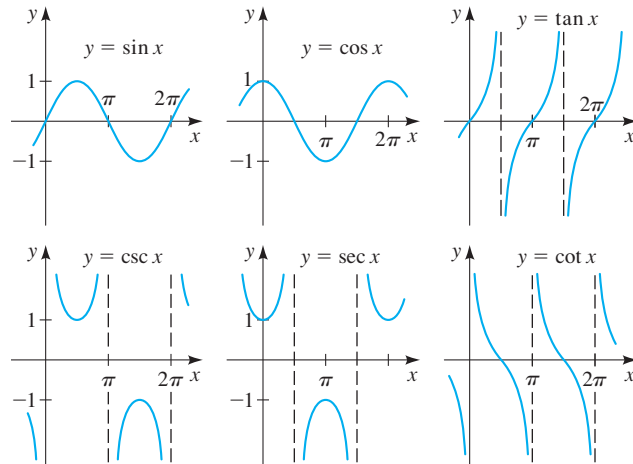
SPECIAL VALUES OF THE TRIGONOMETRIC FUNCTIONS

θ	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—
180°	π	0	-1	0
270°	$3\pi/2$	-1	0	—

SPECIAL TRIANGLES

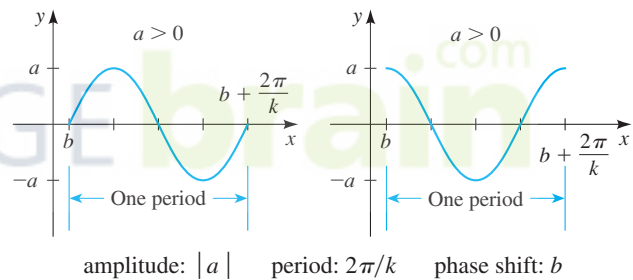


GRAPHS OF THE TRIGONOMETRIC FUNCTIONS

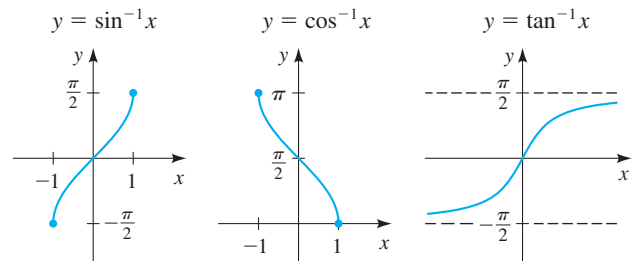


SINE AND COSINE CURVES

$$y = a \sin k(x - b) \quad (k > 0) \quad y = a \cos k(x - b) \quad (k > 0)$$



GRAPHS OF THE INVERSE TRIGONOMETRIC FUNCTIONS



FUNDAMENTAL IDENTITIES

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

REDUCTION IDENTITIES

$$\sin(x + \pi) = -\sin x$$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\cos(x + \pi) = -\cos x$$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$\tan(x + \pi) = \tan x$$

$$\tan\left(x + \frac{\pi}{2}\right) = -\cot x$$

ADDITION AND SUBTRACTION FORMULAS

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \quad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$= 1 - 2 \sin^2 x$$

FORMULAS FOR REDUCING POWERS

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

HALF-ANGLE FORMULAS

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

PRODUCT-TO-SUM AND SUM-TO-PRODUCT IDENTITIES

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

THE LAWS OF SINES AND COSINES

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

